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A VOID GROWTH AND A CYCLIC MODEL IN DUCTILE MATERIAL USING MECHANISM-BASED STRAIN GRADIENT CRYSTAL PLASTICITY THEORY

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ABSTRACT:

This paper addresses the problem of theoretically predicting the evolution of void for a single crystal in ductile material accounting to the size and orientation effects. In this paper, a new damage model is derived based on the theory of mechanism-based strain gradient crystal plasticity (MSG-CP). By imposing the Taylor dislocation model into a widely used Gurson model (1), we extend the Gurson model to account for the void size effect. Meanwhile, we consider the crystal orientation effect by using MSG-CP to describe the behavior of matrix.

Numerical simulation has been conducted under axisymmetric loading condition for cylindrical void and under spherical symmetric tension for spherical void. It reveals that the damage of a ductile porous material has strong orientation-dependence and size-dependence on microscale level. The traditional conclusion that the larger the void size is the faster it grows is also verified by the new model. Additionally, we add a kinematic hardening law to the MSG-CP theory, and have analyzed a hysteresis response of a single crystal under cyclic loading.

Keywords Void size effect; Crystal orientation effect; Gurson model; Taylor dislocation model; Yield condition; MSG-CP theory.

INTRODUCTION

Failure of engineering structures as a result of fracture can lead to catastrophic outcomes. Fracture is caused by damage due to microvoid nucleation, growth, and coalescence in ductile materials. Some physical quantities, such as fracture toughness, can be easily measured through experiments, while others such as incipient damage and damage evolution are often difficult to be measured. Therefore, physically-based theoretical modeling is necessary to complement the experimental research in damage diagnosis and prognosis. Significant efforts are being made to develop experimental and theoretical methods to monitor damage, develop early warning systems, and finally evaluate the remaining life of these engineering structures in order to prevent and/or predict their failure. Recently, health and condition monitoring studies have emerged as a means to monitor the state of structures, allowing for maintenance to be scheduled or other actions to be taken to avoid the consequences of failure, before the failure occurs (2, 3). Prediction and identification of damage type and location, is therefore the core of condition monitoring.

Historically, there are many damage mechanics theories. Some theories were established in a phenomenological framework and others were developed based on the microvoid growth. Numerous theoretical models have been developed in the last three decades to study this dominant failure mechanism in ductile materials. Rice and Tracey (4) investigated the growth of a single void in an infinite matrix, and established that the void growth rate increases exponentially with the
hydrostatic stress $\sigma_{\text{h}}$ imposed on the solid. Gurson (1) adopted a unit-cell model of a single void in a finite matrix, where the volume ratio of the void to the unit cell gives the void volume fraction $f$. In this paper, he established a yield criterion that depends not only on the von Mises effective stress (as in classical plasticity) but also on the hydrostatic stress and void volume fraction $f$. Tvergaard (5) modified the Gurson model to obtain a better agreement with the finite element analyses of void growth, as well as to account for the effect of plastic work hardening.

However, those void models lack two important factors that have strong effects on the materials damage. One is the size effect and the other is the crystal orientation effect. Numerous microscale experiments have shown that materials exhibit strong size effect when the characteristic length is down to microscale (6-17). Although there are some attempts to include the size effect to the damage mechanics theory, the limited size-dependence damage mechanics theories are still based on a phenomenological strain gradient plasticity theory. Recent experimental investigations (18, 19) and numerical studies on microvoids (20-26) have shown that the void growth in ductile materials depends strongly on the void size. The micron- and submicron-sized voids tend to grow slower than larger voids under the same stress level. Without intrinsic material lengths, the classical plasticity theories can not account for the void size effect. Liu et al. (26) investigated the void size effect on the void growth rate based on the Taylor dislocation model (27, 28), which involves an intrinsic material length. For large voids, the void growth rate agrees well with the Rice-Tracey model (4) and displays no size effect. However, for small voids, the void growth rate scales with the square of hydrostatic stress $\left(\sigma_{\text{h}} / \sigma_{\text{y}}\right)^2$, rather than the exponential dependence in the Rice-Tracey model. Here $\sigma_{\text{y}}$ is the tensile yield stress. Wen et al. (29, 30) used the similar approach to extend the Gurson model to solids with cylindrical microvoids and spherical microvoids. These models can show the size-dependence, but the crystal orientation effect has still not been concerned yet.

Some experiments also have shown that the same material with different crystal orientation exhibits different behavior, such as fracture toughness (31, 32). There are many void models for damage. Some consider the void shape effect; some consider the void size effect. But there is no void model considers the orientation effect. In order to incorporate the size and crystal orientation effects to the damage mechanics theory, we propose a new mechanism-based damage mechanics theory.

The proposed damage mechanics theory is based on its entirety on the damage mechanism of ductile materials, namely, microvoid growth and coalescence. More importantly, microvoid growth is directly determined by the specific crystal orientation, as well as microvoid and grain size, in direct contrast to the phenomenological methods approach of existing theories. This new theory is derived by means of the Taylor dislocation model and the recently developed strain gradient crystal plasticity. The crystal orientation and microvoid size effects come into play in the constitutive law of the proposed damage mechanics theory. Given these specific characteristics, the new mechanics-based damage mechanics theory stands distinct from existing theories. The results clearly show an asymmetric loading that has led to unsymmetric deformation due to crystal orientation. This deformation pattern, however, cannot be explained by the existing damage mechanics theories since they do not incorporate crystal orientation information. Additionally, a kinematic hardening law is added to the MSG-CP theory, and hysteretic responses of a single crystal under cyclic loading have been analyzed.

Fundamental inquiry into the damage mechanism for ductile materials is poised to benefit the entire mechanics/materials community. For example, the proposed study on microvoid size and shape effect will play an important role in the design of the hot rolling of aluminum, essential to the aircraft, automobile, manufacturing and consumer products industries.

### CONVENTIONAL CRYSTAL PLASTICITY THEORY

The conventional crystal plasticity theory was developed under a precise mathematical framework by Hill (33), Hill and Rice (34), and Rice (35). For the sake of convenience to reader, the following is just a very brief summary for rate-independent crystal plasticity and limited to small deformation, although the theoretical framework is for both finite and small deformations. The comprehensive review can be found in Asaro and Rice (36, 37). The elastic deformation is ignored in the following formulation since it is negligible compared with plastic deformation.

The plastic strain tensor is associated with the specific slip systems by

$$
\varepsilon = \sum_{\alpha} \gamma^\alpha (s^\alpha \otimes m^\alpha)_{\text{sym}}
$$

where $\alpha$ denotes $\alpha$-th slip system $\gamma^\alpha$ is the plastic shear; $s^\alpha$ and $m^\alpha$ are the slip direction and slip plane normal, respectively; $\| s^\alpha \| = \| m^\alpha \| = 1$ and $s^\alpha \cdot m^\alpha = 0$; $( )_{\text{sym}}$ denotes the symmetrized operation. Here the sum ranges over all activated slip system. Based on the Schmid law (38-40), the shear rate $\dot{\gamma}^\alpha$ of the $\alpha$-th slip system is given by

$$
\dot{\gamma}^\alpha = \frac{1}{h^\alpha} \frac{\dot{\tau}^\alpha}{g^\alpha} \| \gamma^\alpha \| \text{sign}(\gamma^\alpha),
$$

where $\dot{\tau}^\alpha = \tau : (s^\alpha \otimes m^\alpha)$ is the resolved shear stress; $n$ is the rate sensitivity exponent; $g^\alpha$ is the slip resistance that describes the current strength of the system, and its rate is given by $\dot{g}^\alpha = \sum_{\beta} h^\alpha \beta |\dot{\gamma}|$. Here $h^\alpha \beta$ are the slip hardening moduli. There are many existing models that describe the hardening moduli. This includes Asaro’s (37, 38) self hardening moduli, Bassani and Wu’s (41) hardening model. Conventional crystal plasticity has been used to successfully...
investigate some crystal orientation-related problems (39, 40, 42-44).

**TAYLOR DISLOCATION MODEL**

The Taylor dislocation model (27, 28, 45) gives the shear flow stress $\tau$ in terms of the dislocation density $\rho$ by

$$\tau = \alpha \mu b \sqrt{\rho} = \alpha \mu b \sqrt{\rho_S + \rho_G}$$  \hspace{1cm} (2)

where $\mu$ is the shear modulus; $b$ is the Burgers vector; and $\alpha$ is an empirical material constant around 0.3 (27, 28, 46). The dislocation density $\rho$ consists of two parts, namely the density of statistically stored dislocations $\rho_S$ and the density of geometrically necessary dislocations $\rho_G$, where the former is determined from the relation between stress $\sigma$ and plastic strain $\varepsilon$ in uniaxial tension $\sigma = g \rho_S = \sigma_{ref} f(\varepsilon)/M$ and the latter is related to the gradient of plastic deformation by $\rho_G = \bar{\tau} \eta/b$ (47-49). Here $\sigma_{ref}$ is a reference stress (e.g., yield stress $\sigma_Y$); $M$ is the Taylor coefficient; $M = \sqrt{3}$ for an isotropic solid and $M = 3.06$ for a face-centered-cubic (FCC) crystal (50-52); $\bar{\tau}$ is the Nye factor (53) to account for the effect of discrete slip systems on the distribution of geometrically necessary dislocations, and $\eta$ is around 1.9 for FCC crystals (53); and $\eta = 1/2 \sqrt{\eta_{ijk} \eta_{ijk}}$ is the effective strain gradient and $\eta_{ijk} = u_{k,ij}$ is the strain gradient tensor.

Taylor dislocation model has been used to develop strain gradient plasticity theory (47-49), which has successfully explained many micro-scale experiments and phenomena (54-57). In the foregoing strain gradient theories, the constitutive law does not depend on crystal orientation.

Han et al. (58, 59) recently developed a strain gradient crystal plasticity theory and related $\rho_G^S$ to the slip resistance function $g^a$ for $a$-th slip system by $\rho_S^a = \left(g^a / \alpha \mu b \right)^2$ and $\rho_G^a$ to the effective density of geometrically necessary dislocations $\eta_G^a$ by $\rho_G^a = \eta_G^a / b$. The effective density of geometrically necessary dislocations $\eta_G^a$ is given by

$$\eta_G^a = \left\| \sum_{\beta} s^{a\beta} \nabla \gamma^\beta \times \mathbf{m}^\beta \right\|$$  \hspace{1cm} (3)

where $\| \|$ denotes the norm, $s^{a\beta} = s^a \cdot s^\beta$, and the plastic shear $\gamma^\beta$ is related to the macroscopic strain via Eq. (1).

Adding the density of statistically stored dislocations $\rho_S$ and the density of geometrically necessary dislocations $\rho_G$ in Eq. (2) leads to shear stress:

$$\varepsilon^a = g^a \sqrt{\left(g^a / g_0\right)^2 + l \eta_G^a}$$  \hspace{1cm} (4)

where $g_0$ denotes a reference slip resistance and $l$ is an intrinsic length scale associated with strain gradient

$$l = \alpha^2 \mu^2 b \left(g_0^2\right)$$  \hspace{1cm} (5)

Typically, $b$ is around one tenth of a nanometer and $\mu/g_0=100$, and the intrinsic length scale $l$ is estimated to be on the order of a micron, similar to the MSG theory (47, 48, 54, 60). Thus the flow stress can be expressed as

$$\sigma_{flow} = M \tau = Mg_0 \left(\sum_{a} \left(g^a / g_0\right)^2\right) + l \sum_{a} \eta_G^a$$  \hspace{1cm} (6)

The Taylor dislocation model will be also used in the development of the proposed mechanism-based damage mechanics. However, unlike the MSG theory developed by Gao et al. (47) and Huang et al. (48), this new theory will determine the effective strain gradient with respect to specific crystal orientations and the damage mechanism, microvoid growth.

**NEW DAMAGE MODEL ACCOUNTING FOR THE SIZE EFFECT AND ORIENTATION EFFECT**

Nucleation, growth and coalescence of microvoids are common damage mechanisms for ductile materials. The proposed theory will use the microvoid growth rate as the damage index. Unlike existing damage mechanics theories (4) that do not account for the effect of orientation on microvoid growth rate, we determine this important variable directly from strain gradient crystal plasticity (58, 59) and specific crystal orientation. A general procedure is introduced in this section and some results, based on a specific example, are presented.

(1) For a microvoid in an infinite medium and subject to the remote strain field $\varepsilon_0^D$, the strain field in the solid depends on the remote strain field and the geometry of the microvoid (e.g., the microvoid growth rate $D$ and the size and shape of microvoid).

(2) The orientation imaging microscope (OIM) is then used to identify the crystal texture and the activated slip systems ($s^a$ and $m^a$) for the solid.

(3) According to Eq. (1), the plastic shear $\gamma^a$ can be calculated in terms of the remote strain fields and the microvoid growth rate $D$ for each activated slip system obtained in step (2). Thus both the slip resistance $g^a$ and the effective strain gradient $\eta^a$ can be determined depending on the crystal orientation and microvoid growth rate $D$.

(4) Then the flow stress $\sigma_{flow}$ is given by Eq. (6). Once again, the flow stress $\sigma_{flow}$ depends on the microvoid growth rate $D$, i.e.,

$$\sigma_{flow} = M \tau = Mg_0 \left(\sum_{a} \left(g^a (D) / g_0\right)^2\right) + l \sum_{a} \eta_G^a (D)$$

(5) A power-law viscoplastic-limit model is adopted to
link the plastic strain rate \( \dot{\varepsilon}^p \) and the flow stress \( \sigma_{\text{flow}} \) (Eq. 7) by

\[
\dot{\varepsilon}^p = \dot{\varepsilon} \left( \frac{\sigma_e}{\sigma_{\text{flow}}} \right)^n
\]

(8)

where \( \dot{\varepsilon} = \sqrt{\frac{2}{3}} \dot{\varepsilon}_i \dot{\varepsilon}_j \), and \( \dot{\varepsilon}_y = \dot{\varepsilon}_y - \frac{1}{2} \dot{\varepsilon}_{i i} \dot{\varepsilon}_{y i} \) is the deviatoric strain rate; \( \sigma_e = \sqrt{\frac{2}{3}} \sigma_i \sigma_j \) is the von Misses effective stress; \( n \) is a rate-sensitivity exponent, which usually takes a large value (\( \geq 20 \)).

Finally, a mechanism-based damage mechanics theory is developed based on the flow stress \( \sigma_{\text{flow}} \) (Eq. 7) that depends on crystal orientation and microvoid growth rate \( D \). The constitutive law is in the framework of \( J_2 \) flow theory and is given by

\[
\sigma_{ij} = K \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij} + 2 \mu \left[ \dot{\varepsilon}_{ij} - \frac{3 \dot{\varepsilon}^p}{2 \sigma_e} \frac{\sigma_e}{M g_0} (g^T(D)g) + \eta_i \right] \sigma_{ij}'
\]

(9)

This mechanism-based damage mechanics theory incorporates the crystal texture, activated slip systems, microvoid growth rate, and the size effect into the constitutive law. We admit that this proposed damage mechanics theory is preliminary, but it paves the way to study the damage of ductile materials from the fundamental mechanisms. Further development is required to establish a robust and efficient theory.

A CONTINUUM MODEL OF SOLIDS WITH CYLINDRICAL MICROVOID

Here, we show some results for the development of the constitutive law based on the growth of cylindrical microvoids. In the results, we assume that a microvoid has nucleated.

For a cylindrical microvoid of initial radius \( r_0 \) in an infinite medium and subject to remote equi-biaxial tension, \( \varepsilon_{ij}^0 = \varepsilon_{xx}^0 = \varepsilon_{yy}^0 \). If we assume proportional deformation and ignore the elastic deformation (i.e., the solid is incompressible), the non-vanishing displacement field is \( u_x = u_y = u_z = 0 \), where \((r, \theta, z)\) are the cylindrical coordinates; \( u_0 \) is the displacement on the microvoid surface; \( \varepsilon = \sqrt{2 \varepsilon_{xx} \varepsilon_{yy} / 3} = \varepsilon_{xx} / \sqrt{3} \) is the effective strain in the remote field; \( D = u_0 / \sigma_0^V = V / 2 \sigma_0^V \) is the void growth rate under proportional deformation. The non-vanishing strains (also the plastic strain) are given by

\[
\varepsilon_{rr} = -\varepsilon_{\theta \theta} = -\varepsilon_{zz} = -D \frac{\sigma_0^V}{\rho^2}
\]

(10)

In this analysis, we only consider two activated slip system \( s^1 = e_x(\omega), m^1 = e_y(\omega) \), \( s^2 = e_x(\omega + 2\pi/3), m^2 = e_y(\omega + 2\pi/3) \), where \( \omega \) is the polar angle, i.e., the orientation of the first slip system as showed in Fig. 1. According to Eq. (1), the plastic shear \( \gamma^p \) for each slip system can be determined and the effective strain gradient \( \eta^\alpha \) on a slip system \( \alpha \) can then be obtained from Eq. (3),

\[
\eta_0^\alpha = \frac{4}{3} \varepsilon_{\omega}^D \frac{r_0^2}{r} \sin \left( 3\theta - 3\omega + \frac{\pi}{3} \right), \quad \eta_0^\beta = \frac{4}{3} \varepsilon_{\omega}^D \frac{r_0^2}{r} \sin \left( 3\theta - 3\omega - \frac{\pi}{3} \right)
\]

(11)

where \( \theta \) is a polar angle of a material point in the solid. Here we would like to emphasize a unique phenomenon in the analysis. Besides depending on \( \omega \) (orientation of a slip system), the effective strain gradient \( \eta_0^\alpha \) also varies with polar angle \( \theta \), which is different from the mechanism-based strain gradient plasticity theory (48), where the effective strain gradient does not depend on polar angle for the cylindrical microvoid growth. This is because in the proposed theory the equi-biaxial tension in the remote field may not lead to symmetric dislocation slip systems for arbitrary crystal texture, and thus the symmetric loading may not produce symmetric deformation, while the mechanism-based strain gradient plasticity theory smears out the information for crystal orientation via homogenization. The similar unsymmetrical deformation mode due to symmetric loading was also observed in Nemat-Nasser, et al. (61) and numerically verified by Solanki, et al. (32).

By using linear strain hardening in slip resistance

\[
g^\alpha = g_0 + 4\sigma_0^V c_\alpha \varepsilon_{\omega}^D / r^2 \left[ \cos (\theta - \omega - 2\pi/3) + \cos (\theta - \omega + 2\pi/3) - 2\pi/3 \right], \quad \cos (\theta - \omega - 2\pi/3) + \cos (\theta - \omega + 2\pi/3)
\]

the flow stress is then obtained by Eq. (7), where the effective strain gradient \( \eta_0^\alpha \) is given in Eq. (11). It is obvious that the flow stress depends on microvoid growth rate \( D \), the orientation of the slip system via \( \omega_0 \) and the intrinsic length scale \( l \). Among these factors, the size effect has been well addressed (48, 62-64).

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Fig.1 Cylindrical microvoid with two slip systems
In order to address the influence of microvoid growth rate $D$ and the orientation of slip system on the flow stress, we take $M = 3.06$ for FCC crystal, $c_s / g_0 = 0.2$ (59), and $\theta = 2\pi / 3$, $e^\infty = 0.1$, $r_0 / R = 0.5$, $l / r_0 = 1$, and calculate the flow stress for several values of $D$ and $\omega$. It should be pointed out that here we arbitrarily assume a value for the microvoid growth rate $D$, though it depends on remote field and microvoid size. Fig. 2 shows the flow stress $\sigma_{flow} / g_0$ with respect to various values of microvoid growth rate $D$ for given $\omega = \pi / 4$ and $\omega = \pi / 3$, and Fig. 3 gives the flow stress for various values of slip system orientation for given $D = 20$ and 30. It is obvious that both microvoid growth rate $D$ and slip system orientation $\omega$ have very strong effects on the flow stress. Therefore, it is critical to include the crystal orientation information in the constitutive law and the proposed mechanism-based damage mechanics can capture this crystal orientation dependence. This important orientation dependence cannot be studied within the existing framework of damage mechanics.

![Fig.2 Dimensionless flow stress vs microvoid growth rate D](image1)

![Fig.3 Dimensionless flow stress vs slip system orientation ω](image2)

**A CONTINUUM MODEL OF SOLIDS WITH SPHERICAL MICROVOID**

For a spherical void in an infinite medium subjected to remote spherical symmetric tension, $\varepsilon^\infty$. Assume the material is incompressible. The displacement in the radial direction is:

$$u_R = \frac{R^2}{R^2} u_0,$$

where $u_0$ is the displacement on the void surface, $R_0$ and $R$ are the void radius and radial coordinate, respectively. The non-vanishing strain in the spherical coordinates $(R, \theta, \phi)$ and the effective strain are:

$$\varepsilon_R = -2\varepsilon_{\phi\phi} = -2R^2 u_0, \varepsilon = -2\frac{R^2}{R^2} u_0 = \varepsilon^\infty.$$  

The void growth rate under proportional deformation is also defined as: $D = \frac{u_0}{2R}$. So we can get the non-vanishing strain in terms of the effective strain and void growth rate as following $\varepsilon_R = -2\varepsilon_{\phi\phi} = -2\varepsilon_0 R^2 / R^2$. We considered three activated slip systems as showed in fig. 4:

$$\gamma^1 = [\cos \omega, \sin \omega, 0]^T, m^1 = [-\sin \omega, \cos \omega, 0]^T,$$

$$\gamma^2 = [-\cos \omega, \sin \omega, 0]^T, m^2 = [-\sin \omega, -\cos \omega, 0]^T,$$

$$\gamma^3 = (0, 0, 1)^T, m^3 = [\cos \phi, \sin \phi, 0]^T.$$

By using coordinate transformation and Eq. (1), we can get the resolved shear in each slip system:

$$\gamma^1 = \varepsilon D \frac{R^2}{R^2} \left( \frac{B}{\sin 2\omega} + \frac{A}{\cos 2\omega} \right),$$

$$\gamma^2 = \varepsilon D \frac{R^2}{R^2} \left( \frac{A}{\sin 2\omega} + \frac{B}{\cos 2\omega} \right),$$

$$\gamma^3 = -3\varepsilon D \frac{R^2}{R^2} \sin 2\omega \cos \phi,$$

where $\theta = \sin \theta \cos \phi \cos 2\theta$ and $\phi$ are angles of a material point in the solid in spherical coordinates. $\phi$ is the angle between the normal to the third slip plane and $x_1$ axis in Cartesian coordinate system. The effective strain gradient $g^\alpha$ on a slip system can then be obtained from Eq. (3) (the expression for the effective strain gradient is very long and complicated, not presented here).

Substituting the linear relation of slip resistance:

$$g^T = g_0 + c_a \left( |\gamma^1|^2 + |\gamma^2|^2 + |\gamma^3|^2 \right)$$

$$g^T = g_0 + c_b eD \frac{R^2}{R^2} \left( \frac{B}{\cos 2\omega} - \frac{A}{\sin 2\omega} + \frac{A}{\sin 2\omega} + \frac{B}{\cos 2\omega} + \frac{3\sin 2\theta \cos \phi}{\cos \phi} \right).$$

According to Eq. (7), the flow stress accounting for size effect and orientation effect is obtained.
For emphasizing the size effect and orientation effect, we assume $M = 3.06$, $c_b/g_0 = 0.2$, and $\varepsilon^\infty = 0.1$, $r_0/r = 0.5$, $l/r_0 = 1$.

Fig. 6 The flow stress vs spherical coordinate $\theta$

Fig. 7 The flow stress vs spherical coordinate $\phi$

Fig. 5, fig. 6 and fig. 7 show the flow stress has strong dependence on the size of microvoid and the orientation, which is also concluded in cylindrical void analysis. Thus, the proposed constitutive law is key to study the size and orientation effects in damage mechanics.

FATIGUE ANALYSIS

A thorough understanding of micromechanical response and fundamental change mechanism of engineering materials under cyclic loading is essential for microstructural design for fatigue resistance (67). Theses can be done by either experimental observations via electron microscopes or numerical simulations. Based on the MSG-CP theory, we add a nonlinear kinematic hardening rule of the Armstrong-Frederick type, i.e.,
\[ \dot{\chi}^\alpha = h \dot{\gamma}^\alpha - g \chi^\alpha \dot{\gamma}^\alpha \]  
(12)

into the model to conduct the numerical modeling in this study. Where \( h \) and \( g \) are material parameters, \( \dot{\gamma}^\alpha \) is the back stress on the \( \alpha \)th slip system. The corresponding power law is

\[ \dot{\gamma}^\alpha = \gamma_0 \left( \left| \frac{r^\alpha - \chi^\alpha}{G} \right| \right)^n \, \text{sgn}(r^\alpha - \chi^\alpha) \]  
(13)

Following similar approach by Morrissey and Johnston (65, 66), we use the modified MSG-CP theory to describe the material behavior under cyclic loading and conduct the fatigue analysis with a modified ABAQUS UMAT (68). A single crystal copper bar is subjected to fully reversed 24 cyclic loadings controlled by strain, \( \pm 0.003 \), as shown in Fig. 8. Fig. 9 shows the hysteresis response of stress-strain relation to the cyclic loading. It is seen from Fig.5 that stress-strain response shows cyclic strain hardening from 150 MPa to 280 MPa within 12 cycles and then approach a saturation stress at 280 MPa.

CONCLUDING REMARKS

A constitutive equation accounting for size effect and crystal orientation effect have been derived in this paper. The investigated results indicate that the flow stress varies with the microvoid volume growth rate and crystal orientation. It is clearly shown that the crystal orientation effect on flow stress is significant. A kinematic hardening law has been added to the MSG-CP theory and has successfully analyzed a single crystal copper subjected to cycles loading under strain control. The numerical simulating results show single crystal copper cyclic strain hardening and cyclic hysteretic saturation in nature.

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